

Big Bang Nucleosynthesis Constraints on Brane Cosmologies

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We examine constraints from Big Bang nucleosynthesis on type II Randall-Sundrum brane cosmologies with both a dark radiation component and a quadratic term that depends on the 5-dimensional Planck mass, M_5 . Using limits on the abundances of deuterium and helium-4, we calculate the allowed region in the M_5 -dark radiation plane and derive the precise BBN bound on M_5 alone with no dark radiation: $M_5 > 13$ TeV.

In brane cosmologies, the observable universe is a brane embedded in a higher-dimensional bulk. The standard-model fields are confined to our 3-brane, while gravity alone propagates in the bulk. Such models were proposed to explain the large gap between the energy scale of gravity and the energy scales of the other interactions; by adding a number of extra compact dimensions, the “true” Planck mass can be moved down to the TeV scale [1, 2].

An interesting variation on this idea was proposed by Randall and Sundrum [3], who produced a set of models with a non-compactified extra dimension. In the type II Randall-Sundrum model, the brane has positive tension, and the bulk contains a negative cosmological constant. The cosmology produced in this model has been investigated in some detail [4, 5, 6]. With the correct fine-tuning, the Friedmann equation in this scenario reduces to:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3M_4^2}\rho - \frac{k}{a^2} + \frac{\Lambda_4}{3} + \frac{1}{36M_5^6}\rho^2 + \frac{\mathcal{C}}{a^4}, \quad (1)$$

where $M_4 = (8\pi G_N)^{-1/2}$ is the 4-dimensional Planck mass, and M_5 is the 5-dimensional Planck mass. In equation (1), the first three terms are identical to the corresponding terms in the conventional Friedmann equation, corresponding to the contribution from the total density, curvature, and cosmological constant, respectively. (The curvature and cosmological constant have no effect on Big Bang nucleosynthesis, so we will not discuss them further). The final two terms give the modification to the Friedmann equation in this brane scenario; the second of these is sometimes called “dark radiation”, because it scales as a^{-4} ; however, \mathcal{C} can be positive or negative.

Constraints can be placed on both M_5 and \mathcal{C} from Big Bang nucleosynthesis (BBN) [5, 6, 7, 8, 9, 10, 11]. These limits are based on the requirement that the change in the expansion rate due to the quadratic and dark radiation terms in equation (1) be sufficiently small that an acceptable ^4He abundance be produced. With the exception of Ref. [10], these limits are all rather rough. In

Ref. [10], the constraint on the dark radiation term was calculated in detail using limits from both BBN and the cosmic microwave background (CMB), assuming negligible contribution from the quadratic term. (For a further discussion of CMB limits on brane cosmologies, see Ref. [12]).

In this paper, we extend the calculation of Ref. [10] to include both the contribution from the quadratic term and the dark radiation simultaneously, providing a constraint in the M_5, \mathcal{C} plane. This also provides the first detailed calculation of the BBN limit on M_5 alone. Note that M_5 is already constrained by the requirement that the theory reduce to Newtonian gravity on scales > 1 mm; this requirement gives [7]

$$M_5 > 10^5 \text{ TeV}, \quad (2)$$

which is considerably stronger than limits that can be derived from BBN (which are on the order of a few TeV). However, it has been noted [11] that the type II Randall-Sundrum model could be an effective theory, derived from a more fundamental theory, in which case the limit in equation (2) need not apply. It is therefore of interest to consider the BBN limit on M_5 , both alone and in combination with the limit on the dark radiation.

The primordial production of ^4He is controlled by a competition between the weak interaction rates (which govern the interconversion of neutrons and protons) and the expansion rate of the Universe. As long as the weak interaction rates are faster than the expansion rate, the neutron-to-proton ratio (n/p) tracks its equilibrium value. Eventually, as the Universe expands and cools, the expansion rate comes to dominate and n/p essentially freezes out. The relatively large binding energy of ^4He insures that nearly all the neutrons which survive this freeze-out are converted into ^4He as soon as deuterium becomes stable against photodisintegration (see, e.g., Ref. [13] for further details of this well-known story). In the standard cosmology [i.e., one where the Friedmann equation takes the form $(\dot{a}/a)^2 \sim \rho/(3M_4^2)$], n/p freezes out at a temperature $T \sim 1$ MeV. Therefore, the primor-

dial production of ^4He is very sensitive to the expansion rate of the Universe at temperatures ~ 1 MeV, and in fact this sensitivity has been exploited many times in bounding the number of light neutrino species.

Our analysis uses the sensitivity of primordial ^4He to the expansion rate at $T \sim 1$ MeV to constrain the two additional “brane-terms” that appear in the type II Randall-Sundrum generalization of the Friedmann equation (equation 1). Specifically, we limit the additional energy density associated with the M_5 and dark radiation contributions by requiring that the primordial production of ^4He in such brane cosmologies agree with the observed abundance of ^4He . Note that the primordial production of ^4He is weakly (log) dependent on the baryon density and so in principle constraints on such brane cosmologies would also involve constraints on the baryon density. However, unlike ^4He , the abundance of deuterium is very sensitive to the baryon density and fairly insensitive to the expansion rate (which we verify in our detailed calculations). Therefore, we can use a comparison of the predicted deuterium abundance and its observed abundance to independently constrain the baryon density and then examine the subsequent constraints on M_5 and the dark radiation coming from ^4He .

In order to constrain the additional brane-terms, we start with a standard BBN model and some conservative estimates of the primordial abundances of the light elements. We assume 3 light neutrino species and the standard updated nuclear reaction network. Because the dark radiation scales as a^{-4} , it is convenient to parametrize it in terms of the effective number of additional neutrino species, ΔN_ν , given by $\Delta N_\nu \equiv (\mathcal{C}/a^4)(3M_4^2/\rho_\nu)$, where ρ_ν is the energy density contributed by a single, two-component massless neutrino, and ΔN_ν can be either positive or negative. (Note that a slightly different parametrization was used in Ref. [10]). Then the brane Friedman equation can be completely specified by M_5 and ΔN_ν .

We take the primordial deuterium abundance as inferred from QSO absorption line systems in the range

$$3 \times 10^{-5} < \text{D}/\text{H} < 4 \times 10^{-5}, \quad (3)$$

and we take Y_p (the primordial mass fraction of ^4He) as inferred from low-metallicity HII regions to be

$$0.23 < Y_p < 0.25, \quad (4)$$

(see, e.g., Ref. [13]). For a fixed pair of M_5 and ΔN_ν , we then scan over the baryon-photon ratio η to see if a value of η exists which produces deuterium and ^4He abundances within the acceptable limits. In practice, the deuterium abundance is nearly insensitive to M_5 and ΔN_ν , so the deuterium constraint limits η to the range $\eta = 4 - 6 \times 10^{-10}$ for most of our parameter range, and the upper and lower limits on ^4He then give the constraints in the M_5 – dark radiation plane (although note that our procedure is “exact” in the sense described above; we do not assume a fixed bound on η). Our results are shown

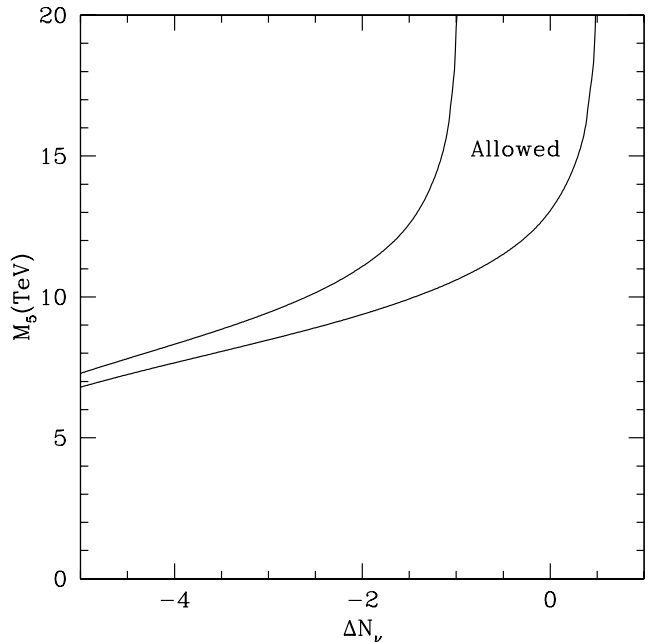


FIG. 1: The area between the two curves gives the region in the M_5 –dark radiation plane allowed by Big Bang nucleosynthesis, where M_5 is the 5-dimensional Planck mass, and the dark radiation is parametrized in terms of ΔN_ν , the effective number of additional two-component neutrinos. In terms of the quantities appearing in the brane-cosmology Friedman equation, $\mathcal{C}/a^4 = (\Delta N_\nu \rho_\nu)/(3M_4^2)$, where ρ_ν is the energy density of a single two-component neutrino.

in Fig. 1, where we indicate the region in the M_5 – dark radiation plane consistent with our adopted range of primordial abundances. (All the brane-cosmologies in our allowed region produce acceptable primordial ^7Li abundances.)

We can understand the shape of the excluded region shown in Fig. 1 as follows. The left and right contours of the allowed region correspond to $Y_p = 0.23$ (lowest allowed expansion rate) and $Y_p = 0.25$ (largest allowed expansion rate), respectively. The sensitivity of primordial ^4He to the expansion of the Universe at $T \sim 1$ MeV constrains the additional energy density at that temperature to be within roughly half a neutrino species of the standard model. When M_5 is sufficiently large, we get the standard constraint on additional relativistic energy density, which appears in Fig. 1 as a vertical band. As M_5 decreases, the ρ^2 term in the Friedmann equation starts to dominate at $T \sim 1$ MeV and our allowed region moves sharply to the left, so that the negative dark radiation energy density cancels the ρ^2 contribution to the expansion.

Our investigation yields two new results: a precision determination of the BBN limit on M_5 , and a combined limit on M_5 and \mathcal{C} . For $\mathcal{C} = 0$, we derive the precise BBN

limit on M_5 alone:

$$M_5 > 13 \text{ TeV.} \quad (5)$$

Our limits on the dark radiation for large M_5 are

$$-1.0 < \Delta N_\nu < 0.5, \quad (6)$$

in rough agreement with Ref. [10]. When both the quadratic term and the dark radiation contribute significantly to the Friedman equation, our limits are given by the contours in Fig. 1. Although we can find acceptable solutions for very small M_5 , the allowed region shrinks

dramatically in this limit; a small M_5 requires both $\mathcal{C} < 0$ and precise fine-tuning of \mathcal{C} .

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- [1] N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. B **429**, 263 (1998); Phys. Rev. D **59**, 086004 (1999).
 - [2] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. B **436**, 257 (1998)
 - [3] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999); Phys. Rev. Lett. **83**, 4690 (1999).
 - [4] C. Csaki, M. Graesser, C. Kolda, and J. Terning, Phys. Lett. B **462**, 34 (1999).
 - [5] J.M. Cline, C. Grojean, and G. Servant, Phys. Rev. Lett. **83**, 4245 (1999).
 - [6] P. Binetruy, C. Deffayet, U. Ellwanger, D. Langlois, Phys. Lett. B **477**, 285 (2000).
 - [7] R. Maartens, D. Wands, B. Bassett, and I. Heard, Phys. Rev. D **62**, 041301 (2000).
 - [8] D. Langlois, R. Maartens, M. Sasaki, and D. Wands, Phys. Rev. D **63**, 084009 (2001).
 - [9] J.D. Barrow and R. Maartens, Phys. Lett. B **532**, 155 (2002).
 - [10] K. Ichiki, M. Yahiro, T. Kajino, M. Orito, and G.J. Mathews, [arXiv:astro-ph/0203272].
 - [11] S. Mizuno and K. Maeda, [arXiv:hep-ph/0108012].
 - [12] B. Leong, A. Challinor, R. Maartens, and A. Lasenby, [arXiv:astro-ph/0208015].
 - [13] K.A. Olive, G. Steigman, and T.P. Walker, Phys. Rep. **333**, 389 (2000).